Magnetohydrodynamic Energy Conversion

Magnetohydrodynamic energy conversion, popularly known as MHD, is another form of direct energy conversion in which electricity is produced from fossil fuels without first producing mechanical energy. The process involves the use of a powerful magnetic field to create an electric field normal to flow of an electrically conducting fluid through a channel as depicted in Figure 2. The flow velocity is parallel to the channel axis, taken in the y-direction. The drift of electrons induced by this lateral electric field produces an electric current density vector **J**. Electrodes in opposite side walls of the MHD flow channel provide an interface to an external load, to the MHD channel flow to the electrode on the opposite wall, and then back to the fluid, completing a circuit. Thus the MHD channel flow is a direct current source that can be applied directly to an external load or can be linked with a power-conditioning inverter to produce alternating current.

MHD effects can be produced with electrons in metallic liquids such as mercury and sodium or in hot gases containing ions and free electrons. In both cases, the electrons are highly mobile and move readily among the atoms and ions while local net charge neutrality is maintained. That is, while electrons may move with ease, any small volume of the fluid contains the same total positive charges on the ions and negative electron charges, because any imbalance would produce large electrostatic forces to restore the balance.

Ionized Gases in Electromagnetic Fields

Before analyzing the MHD channel, we will consider briefly the behaviour of electrons in an ionized gas in the presence of electromagnetic fields.

In a gas at or near equilibrium, atoms, ions, and elections are in random motion. At any given spatial position their velocities are distributed about a mean velocity that increase with increase in the local temperature. Consider just one of the free electrons moving, without collision, in a plane normal to a uniform magnetic field, as in Figure 1 below. The electron experiences a constant force qc_cB normal to its path. Here, q is the charge of the electron and c_c the magnitude of its velocity. Because the force is normal to its path, the electron travels with constant velocity on a circular path around magnetic lines of force. By Newton's Second Law, the force on the electron is

$$F = m_e c_e^2 / r = qc_e B \qquad [N] \qquad (1)$$

It follows that the angular frequency of the electron about a line of force c_c/r, called its *cyclotron frequency,* is

$$\omega = c_e / r = qB / m_e \qquad [s^{-1}] \qquad (2)$$

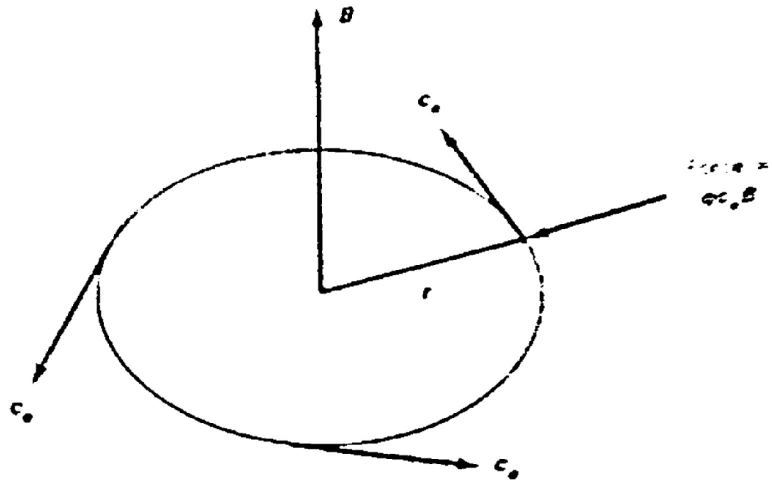

Fig. 1

The electron cyclotron frequency is independent of electron velocity and is dependent only on the magnetic field strength and electron properties. Although the cyclotron motions of electrons exist in gases when strong magnetic fields are present, the circular paths of the electrons may be disrupted by collisions with other parties.

The likelihood of collisions between particles depends on their effective sizes: larger particles will collide more frequently. The probability of collision is taken as proportional to the collision cross-section Q of the particle, which may be thought of as its area. The frequency of collision of electron ω_c is given by the product of the electron number density, n_e (electrons/m^3), the collision cross-section, Q [m^2], and the velocity, c_e [m/s]:

$$\omega_c = n_e Q c_e = 1/\tau \qquad [collisionss\ per\ s] \qquad (3)$$

Here, the mean time between Collins, τ[s], is the inverse of the collision frequency.

The ratio of the cyclotron frequency to the collision frequency ω/ω_c, is called the *Hall parameter*. It indicates the relative importance of the magnetic field and collisions in controlling electron motion in the ionized n gas. The Hall parameter is related to the magnetic field intensity by

$$\omega/\omega_c = \omega\tau = qB\tau/m_e = qB/n_e m_e Q c_e \qquad (4)$$

It is proportional to the number of cyclotron loops made collision. A Hall parameter large compared with one indicates magnetic-field-dominated motion of electrons, while a small value implies that collisions quickly break up ordered motions produced by the magnetic field.

At least three velocities are of importance in a conducting gas in a MHD channel. First, the velocity of the gas stream is given by **u** (assumed constant for the present case for an appropriately designed channel). Secondly, the velocities of individual electrons **c**$_e$, as just introduced, are distributed about an average value that increases with the local temperature. In the absence of electromagnetic fields, the average value of c_c over all electrons is the flow velocity **u**; i.e., on the average the electrons move with the gas flow. When fields are present, however, there may be an average motion of electrons relative to the gas. The third velocity, the relative velocity of an electron **w**$_c$, is defined as the vector difference of its absolute velocity and the mean fluid velocity:

$$\omega_e = c_e - u \qquad [m/s] \tag{5}$$

The drift velocity w_e, is the magnitude of the average of the relative velocities of the electrons. In the absence of fields, the average of **c**$_e$ is **u**, and thus the drift velocity is zero. When an electric field is present, however, the transport of negative charge by electrons represents a current flow in the gas.

Another important parameter, *the electron mobilityμ,* is a measure of the response of electrons to an electric field. It is defined as the ratio of the magnitude of the electron drift velocity w_e to the local electric field intensity.

$$\mu = w_c / E \qquad [m^2/V-s] \tag{6}$$

If it is assumed that an electron loses all of its drift velocity on collision, the acceleration of the electron may be approximated by the ratio of the drift velocity to the mean time between collisions. Because the force due to the electric field is given by qE, Newton's School Law allows the drift velocity to be expressed as

$$w_c = qE\tau / m_e \qquad [m/s] \tag{7}$$

The electron mobility can then be written as

$$\mu = q\tau / m_e \qquad [m^2/V-s] \tag{8}$$

Using equation (4), the product μB becomes the Hall parameter:

$$\mu B = q\tau B / m_e = \omega\tau = \omega / \omega_c \tag{9}$$

Thus the Hall parameter is large for gases of high electron mobility in strong magnetic fields. It will be seen that this can have a significant effect on MHD channel design.

Assuming electrons as the dominant charge carriers, the current density can also be related to the electron mobility through the drift velocity:

$$J = n_e q w_e = n_e q^2 \tau E / m_c \tag{10}$$

$$= \mu n_e q E \qquad [A/m^2]$$

The electron conductivity of a stationary gas is then given by (σ):

$$\sigma = J/E = \mu n_c q \qquad [(\Omega - m)^{-1}] \tag{11}$$

Thus high electron mobility and electron number density are essential to achieve the high conductivity needed in an MHD generator.

Analysis of a Segmented Electrode MHD Generator

Consider the one dimensional flow of a gas in an MHD channel coupled with a simple three-dimensional model of the electromagnetic phenomena. Rather than the continuous electrode configuration , we examine a refined configuration, shown in Figure 2 . we examine a configuration, shown in Figure 3. Here the electrodes, set in opposite electrically insulated channel walls, are segmented in the stream wise direction. This eliminates a return path along the wall for axial electrical currents in the flow.

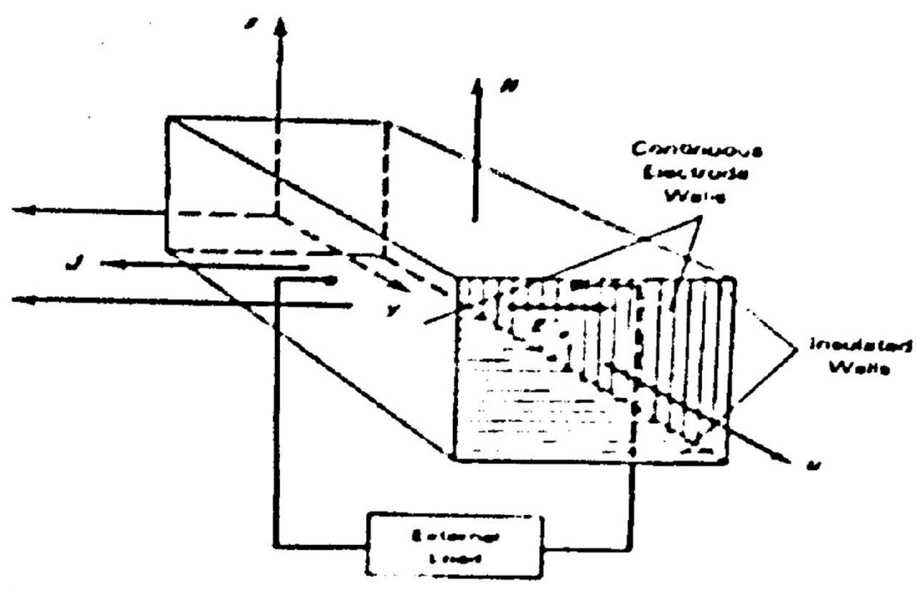

Fig. 2

By the same notation as in Figure 2, a seeded, ionized gas flows through the segmented-electrode channel in the y-direction with a constant velocity *u.* A uniform magnetic field in the z-direction exists throughout the gas in the channel. A force given by q**uxB**, and thus an equivalent electric field **uxB** is imposed on the flow in the channel. Therefore, positive ions tend to drift in the positive x-direction and electrons drift in the negative x-direction toward the right electrodes. Because their mobility is much greater than that of the relatively massive ions, the electrons are the primary charge carriers. The electrons are collected at the right electrodes and flow through the external circuits returning to the channel at the left electrodes, as shown in Figure 3.

When the channel is under an electrical load, the current density vector in the .t-direction induces a force on the fluid in the negatives-direction. Thus the x-component of J interacts with the magnetic field to produce the axial electric field component E_y= - JB that opposes the flow velocity *u.* In order to maintain a constant velocity in the duct, a streamwise pressure gradient, *dp/dy,* must balance the force due to this axial electric field

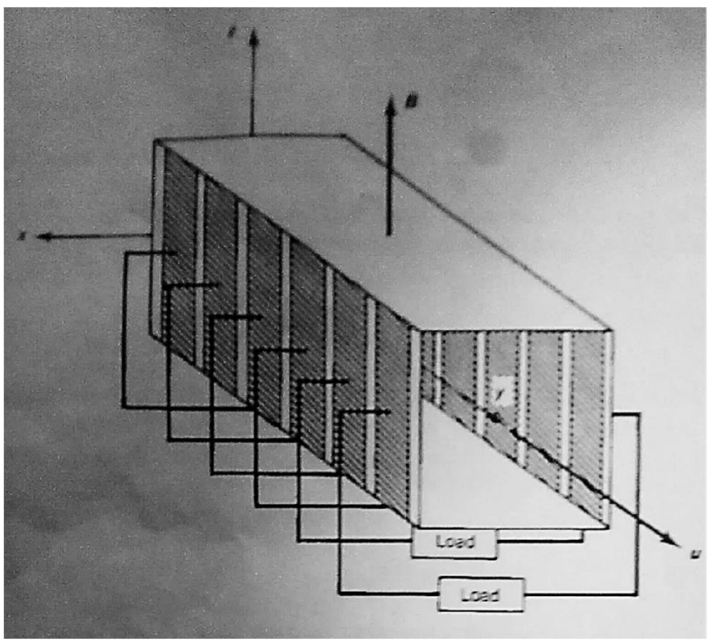

Fig. 3

and the viscous forces. Thus, ignoring viscous resistance, the axial force on the gas per unit volume is

$$F_y = -/J \times B/ = -JB = dp/dy \quad \left[N/m^3\right] \tag{12}$$

where the negative sign indicates that the magnetic force is directed upstream. As a result *dp/dy* < 0, indicating that the flow pressure drops as y increases. The resulting net pressure force in the positives-direction balances the magnetic and viscous forces and maintains the flow velocity constant. A compressor is therefore required upstream of the channel to pressurize the flow, to support the field-induced streamwise pressure gradient, and thus to maintain the steady flow in the channel.

With segmented electrodes there is no axial current in the channel *(Jy* = 0), and thus the current density component J_x=J is proportional to the net electric field in they direction;

$$J = \sigma(uB - E_x) \qquad [A/m^2] \tag{13}$$

The combined electrical resistance of the MHD channel flow and the external load governs the available potential at the MHD electrodes. If the external circuit is open, *J*= 0; hence, Equation (14) indicates that $E_x\big|_{open} = uB$. With a finite external resistance, current flows and the electrode potential is reduced below the open-circuit value. Thus, under load, the channel voltage drops to a fraction *K* of the open-circuit voltage. Hence, we may write $E_x = KuB$, where *K* is called the *channel load factor* and where $0 \le K \le 1$. The current density then becomes

$$J = \sigma uB\ (1 - K) \qquad [A/m^2] \tag{14}$$

The electrical power delivered to the load per unit volume of channel is then given by

$$Power\big|_{out} = \mathbf{J}.E = \sigma u^2 B^2 K\ (1 - K) \qquad [W/m^3] \tag{15}$$

Returning now to consideration of the stream wise electrical fluid interaction, we write the steady-flow form of the First Law for an adiabatic control volume, including the work done against the body force, as

$$m(h_1 + u_2^1/2) = m\ (h_2 + u_2^2/2) + mw \qquad [J/s] \tag{16}$$

where *m* is the channel mass flow rate. Work is positive here because it is done by the fluid in the channel to produce the electrical current flow to the external load. For constant velocity in the channel, this becomes

$$\begin{aligned} h_2 &= h_1 - w = h_1 - \left(Power\big|_{out}\right)(Volume)/m \\ &= h_1 - \sigma u^2 B^2 K\,(1-K)/\rho \qquad [kJ/kg] \end{aligned} \tag{17}$$

where ρ is the gas density. Thus the work delivered to the load reduces the thermal energy of the flow. We have seen that a compressor is required to pressurize the flow in the channel and that heating of the flow provides a high entrance enthalpy and work output.

From Equations (12) and (14), the electrical retarding force on the flow is

$$F_y = \sigma u B^2 (1-K) \qquad [N/m^3] \tag{18}$$

and the fluid power to push the gas through the channel per unit volume is

$$Power|_{in} = |F_y u| = \sigma u^2 B^2 (1-K) \qquad [W/m^3] \tag{19}$$

The Ohmic or I^2R loss is given, using Equation (14), by

$$\begin{aligned} J^2/\sigma &= [\sigma u B(1-K)]^2/\sigma = \sigma(\sigma B)^2(1-2K+K^2) \\ &= \sigma u^2 B^2 (1-K) - \sigma u^2 B^2 K(1-K) \qquad [W/m^3] \end{aligned} \tag{20}$$

Comparing equations (15) and (19) with equation (20), we see that the ohmic loss is the difference between the power required to push the flow through the channel and the useful power through the load.

The efficiency of the channel is defined as the ratio of the $Power|_{out}$ to $Power|_{in}$. By Equations (15) and (19), the MHD channel efficiency is

$$\eta = Power|_{out} / Power|_{in} = K \tag{21}$$

Thus the electrical efficiency of the segmented electrode MHD channel is equal to the channel load factor. Examination of Equation (15) shows that the power output vanishes when $K = 0$ and when $K=1$. Thus there must be an intermediate value of K that maximizes the power output. By differentiation of Equation (15) with respect to K, the usual methods of calculus indicate that the power is maximized when K = 0.5. Thus operation at this value implies that 50% of the flow energy input to the channel is converted to electricity, and the remainder is dissipated in the flow channel. This energy is not lost from the flow but is an irreversibility that is reflected in a loss in ability of the flow to do work.

EXAMPLE I 1.6

A 10 m^3 MHD generator with segmented electrodes has a short-circuit current density of 12,000 amperes per square meter. The gas conductivity is 20 $(\text{Ohm-m})^{-1}$. If flow and magnetic field conditions are unchanged when the load factor is 0.6, what is the output power? What is the actual current density in the channel? If the magnetic field is doubled in strength, by what factor would you expect the output power to change?

Solution

For a short-circuit condition, the load factor is zero, and Equation (14) yields

$$J_{sc} = \sigma u B = 12000 \ A/m^2$$

Then

$$uB = J_{sc} / \sigma = 12000/20 = 600 V/m$$

The power output is then given by Equation (15):

$$Power = \sigma u^2 B^2 K(1-K)V = 20(600)^2 (0.6)(0.4)(10) = 17{,}280{,}000 \ W = 17.28 MW$$

The channel current density is then given by

$$J = \sigma u B (1-K) = 20(600)(0.4) = 4800 \ A/m^2$$

If the magnetic field strength is doubled. Equation (15) shows that the power output is increased by a factor of four, assuming there is no change in the flow or load conditions.